TM-70-1022-11

TECHNICAL MEMORANDUM

DEFLECTION OF FLEXIBLE BALLAST BEAMS ON A SPINNING SPACECRAFT

Bellcomm

BELLCOMM, INC.
955 L'ENFANT PLAZA NORTH, S.W., WASHINGTON, D.C. 20024

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ABSTRACT

An artificial gravity experiment is one of several options being considered for the proposed second Skylab. In order to obtain acceptable rotational dynamic behavior, modifications to the present Skylab configuration are required to meet necessary mass property relationships. One possible method to improve mass properties is the addition of ballast on deployable beams. Recent simulations of vehicle spin-up have shown that large deflections of the ballast beams, leading to structural failure, can occur. This memorandum investigates the case of structural instability of ballast beams on a spacecraft undergoing steady rotation. A method of estimating ballast beam stiffness requirements is presented. Steady rotation imposes no stiffness requirements on the beam configuration under consideration for Skylab.

Beam stiffness requirements for stability for Skylab arise therefore from non-steady rotation conditions. These conditions are briefly discussed.

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SUBJECT: Deflection of Flexible Ballast Beams on a Spinning Spacecraft

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FROM: R. J. Ravera

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TECHNICAL MEMORANDUM

1.0 INTRODUCTION

An artificial gravity mission option is being considered for the proposed second Skylab. The artificial g field is to be obtained by spinning the vehicle about its mass center. Assuming that the planar solar arrays are, as in the first Skylab, perpendicular to the vehicle z-axis (See Figure 1), it is then desirable that the vehicle z-axis be the spin axis and that it be aligned parallel to the solar vector. A recent study(1) has shown that during spin up and constant rotation, it is not possible to align the z-axis to within some small angle ($\sim 5°$) of the solar vector without modification to the present Skylab configuration. The use of ballast on deployable beams is one suggested method for improving mass properties so that alignment can be held within limits.(1) However, dynamic simulations of spin-up revealed that deformations of the ballast beams became large enough to cause structural instability.

Potential sources of instability arise in both steady and non-steady rotation. This memorandum examines the case of steady rotation and derives conditions on beam stiffness for stability. The non-steady rotation case is briefly discussed.

2.0 EQUATIONS OF MOTION OF FLEXIBLE BEAMS ON A SPINNING SPACECRAFT

The analysis will be based on the general model shown in Figure 2. The counterweights (ballast) can be displaced from the mass center of the spacecraft in three directions and it will be assumed that components of angular velocity can exist along all three axes. The mass properties of the beams are identical; therefore, due to symmetry, only one of the ballast beams need be considered. Since the instability is associated with flexural deflections, we need not write the equation of motion in the beam's axial (y) direction. The flexural equations of motion are adapted from more general equations derived by Likens.(2) The following quantities are defined:

$K_{x,z}$ - effective flexural stiffness coefficient in x, z direction

m_b - mass of beam

m_{cw} - mass of counterweight

$q = \begin{bmatrix} q_x \\ q_y \\ q_z \end{bmatrix}$ - deflection vector of counterweight

$r = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$ - undeflected position vector from spacecraft center of mass to counterweight

$\omega = \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix}$ - angular velocity vector of spacecraft

If it is assumed that the total mass of the spacecraft is much greater than the mass of the beam and counterweight, the resulting flexural equations for deflection of the counterweight in a torque free environment are

$$\ddot{q}_x + \left[\frac{K_x}{m} - (\omega_y{}^2 + \omega_z{}^2) \right] q_x = \dot{\omega}_z \; [Y+q_y] - \dot{\omega}_y \; [Z+q_z]$$

$$+ \omega_z[2\dot{q}_y - (Z+q_z)\omega_x] - \omega_y[2\dot{q}_z + (Y+q_y)\omega_x] + [\omega_y{}^2+\omega_z{}^2]X \quad (1)$$

and

$$\ddot{q}_z + \left[\frac{K_z}{m} - (\omega_y{}^2+\omega_x{}^2)\right] q_z = \dot{\omega}_y [X+q_x] - \dot{\omega}_x [Y+q_y]$$

$$+ \omega_y [2\dot{q}_x - (Y+q_y)\omega_z] - \omega_x [2\dot{q}_y + (X+q_x)\omega_z] + [\omega_x{}^2+\omega_y{}^2] Z \qquad (2)$$

where the mass term m,

$$m = m_{cw} + \frac{33}{140} m_b \quad ,$$

is the equivalent tip mass for a massless beam in flexure. An obvious cause of large deflections is a large forcing function (right hand side of equations (1) and (2)). The more subtle forms of instability are:

1) Steady state; the coefficient of the q_x or q_z term in (1) and/or (2) goes to zero indicating a zero restoring force.

2) Non-steady; the time dependence of the coefficients of q_x and q_z can lead to instabilities (large deflections and subsequent failure).[3]

3.0 EFFECTIVE STIFFNESS COEFFICIENTS

The stiffness coefficients, K_x and K_z, must properly take into account the effects of axial (tensile) loads on the transverse deflections of a beam. The axial loads are inertia loads due to the spinning motion of the spacecraft and are centrifugal if the angular rate is constant. It is this effect which gives the apparent stiffness to a spinning string-mass system. It can be shown that for the beam pictured in Figure 3, the relationship between transverse load P, axial load Q, and transverse deflection x is[4]

$$P = \frac{Q}{L} \left(1 - \frac{\tanh \rho L}{\rho L}\right)^{-1} x$$

where

$$\rho^2 = Q/EI$$

A good approximation to this formula is[5]

$$P \approx \frac{3EI}{L^3}\left[1 + \frac{1}{3}(\rho L)^2\right] x$$

$$= \left[\frac{3EI}{L^3} + \frac{Q}{L}\right] x$$

$$= Kx. \qquad (3)$$

Note that

$$K^1 = \frac{3EI}{L^3}$$

is the usual flexural stiffness coefficient for a non-rotating cantilever beam.

4.0 TRANSVERSE, STEADY STATE DEFLECTIONS

The exact solution of the non-steady problem requires the solution of (1) and (2) coupled with an analysis of total vehicle motion. This study will concentrate on the steady state instability; some aspects of the non-steady problem will be discussed later. The steady state deflections of the counterweight are from (1) and (2)

$$q_x = \frac{(\omega_y^2+\omega_z^2)X - \omega_x\omega_y(Y+q_y) - \omega_x\omega_z(Z+q_z)}{\left[\frac{K_x}{m} - (\omega_z^2+\omega_y^2)\right]} = \frac{N_x}{D_x} \qquad (4)$$

and

$$q_z = \frac{(\omega_x^2+\omega_y^2)Z - \omega_x\omega_z(X+q_x) - \omega_y\omega_z(Y+q_y)}{\left[\frac{K_z}{m} - (\omega_x^2+\omega_y^2)\right]} = \frac{N_z}{D_z} \qquad (5)$$

Clearly, when the denominators of (4) and (5), D_x and/or D_z, become zero, unbounded deflections result. A negative denominator indicates a restoring force which augments rather than opposes deflection, also a clear case of structural instability. The study of the structural instability of the beam under steady state conditions therefore boils down to the study of D_x and D_z.

As noted in the last section, the effective beam stiffness depends on the axial load. The axial load in the beam due to inertia effects can be found from

$$Q = m^* \left[(\omega_z^2+\omega_x^2) Y - \omega_y \omega_z Z - \omega_x \omega_y X \right] \tag{6}$$

where the mass term m^*,

$$m^* = m_{cw} + 1/2\ m_b \quad ,$$

includes the fraction of beam mass to be lumped with the counterweight. Noting from Figure 2 that the length of the beam is L=Y, it follows from (3) and (6) that

$$\frac{K_z}{m} = \frac{K_x}{m} = \frac{3EI}{Y^3 m} + \frac{m^*}{m} \left[(\omega_z^2+\omega_x^2) - \omega_y \omega_z (\tfrac{Z}{Y}) - \omega_x \omega_y (\tfrac{X}{Y}) \right] \tag{7}$$

The term $3EI/Y^3m$ corresponds to the square of the first circular frequency of a like cantilevered beam in a non-rotating environment; that is

$$\omega_n = \left(\frac{3EI}{Y^3 m} \right)^{1/2} \tag{8}$$

Combining (7) and (8) with (4) and (5) gives

$$D_x = \omega_n^2 + \frac{m^*}{m} \left[(\omega_z^2+\omega_x^2) - \omega_y \omega_z (\tfrac{Z}{Y}) - \omega_x \omega_y (\tfrac{X}{Y}) \right] - \omega_y^2 - \omega_z^2 \tag{9}$$

and $$D_z = \omega_n^2 + \frac{m^*}{m} \left[(\omega_z^2+\omega_x^2) - \omega_y \omega_z (\tfrac{Z}{Y}) - \omega_x \omega_y (\tfrac{X}{Y}) \right] - \omega_x^2 - \omega_y^2 \tag{10}$$

As stated in the introduction, the z-axis is the desired spin axis, but it is known that stable steady rotation is possible only if the rotation axis is the axis of maximum moment of inertia (see Reference 1). The purpose of the beam and counterweights is in fact to align this principal axis with the z-axis. But given imperfect alignment, there will be some constant components of angular velocity along the x and y axes. It is convenient to write

$$\omega_x = \alpha\omega_z$$

and

$$\omega_y = \beta\omega_z$$

where

$$-1.0 \leq \alpha,\beta \leq 1.0 \quad .$$

Equations (9) and (10) can then be written

$$D_x = \omega_n^2 - \omega_z^2 \left[\beta^2 + \left(\frac{m^*Z}{mY}\right)\beta + \left(\frac{m^*X}{mY}\right)\alpha\beta + 1 - \frac{m^*}{m}(1+\alpha^2)\right] \tag{11}$$

and

$$D_z = \omega_n^2 - \omega_z^2 \left[\beta^2 + \left(\frac{m^*Z}{mY}\right)\beta + \left(\frac{m^*X}{mY}\right)\alpha\beta + \alpha^2 - \frac{m^*}{m}(1+\alpha^2)\right] \tag{12}$$

An estimate of the required ballast beam natural frequency can be obtained from (11) or (12) by noting that in order to satisfy

$$D_x > 0$$

and

$$D_z > 0$$

we must have

$$\omega_n > \omega_z \left[\beta^2 + \left(\frac{m^*Z}{mY}\right)\beta + \left(\frac{m^*X}{mY}\right)\alpha\beta + 1 - \frac{m^*}{m}(1+\alpha^2)\right]^{1/2} \tag{13}$$

and $$\omega_n > \omega_z \left[\beta^2 + \left(\frac{m^*Z}{mY}\right)\beta + \left(\frac{m^*X}{mY}\right)\alpha\beta + \alpha^2 - \frac{m^*}{m}(1+\alpha^2)\right]^{1/2} \tag{14}$$

The expression yielding the highest value of ω_n sets the limit; since it is not expected that α will ever approach unity, equation (13) will provide the limiting value. A zero or imaginary value for required natural frequency indicates that centrifugal force alone is sufficient for providing restoring forces.

5.0 SOME GENERAL RESULTS

A few general conclusions can be drawn from equation (13). First, since $m^* > m$, it follows that if $\beta = 0$ ($\omega_y = 0$), no inherent stiffness is required to provide bounded deformations. Reduction of off-set mounting positions Z and X reduce structural requirements for the beam. If the X off-set distance is eliminated, the required stiffness is independent of the sign of α. Finally note that with $X = 0$, increasing $|\alpha|$ reduces required ω_n and hence, required stiffness; however, this is undesirable from the attitude point of view since increasing $|\alpha|$ means increasing $|\omega_x|$.

6.0 NUMERICAL RESULTS

Dynamic simulations of Skylab spin-up with ballast beams have been conducted at Bellcomm. The ballast beam configuration used in these studies has the following properties:

$m_b = 6$ slugs,

$m_{cw} = 10$ slugs,

$X = 0$,

$Y = 100$ ft,

and $Z = 10$ ft.

The cross section properties of the beam are those of a de Havilland STEM(6) (Storable Tubular Extendible Member) with a 5 inch diameter, 0.025 inch wall and a 43% overlap. The flexural rigidity for such a beam is

$$EI = (29. \times 10^6)\ (1.755)\ \text{lb-in}^2 = 50.891 \times 10^6\ \text{lb-in}^2$$

From (8), the natural cantilever frequency of this ballast beam system is computed to be

$$\omega_n = 0.093 \text{ rad/sec} = 0.015 \text{ cps}$$

Based on this value of ω_n, we can compute ratios of ω_n/ω_z for three possible values of the spin frequency; ω_z = 4, 6 and 8 RPM. These ratios are plotted as horizontal lines on Figure 4. In addition, Fig. 4 contains a plot of the required ratio of ω_n/ω_z for bounded deflections versus values of the parameters α and β. At 8 RPM, the beam used in the Skylab study will reach structural instability under steady rotation if β exceeds 0.3, with α = 0. A β of 0.3 is equivalent to a 17° offset of the axis of maximum moment of inertia from the geometric Z-axis in the Y-Z plane. However, it is expected that the beams and counterweights will limit this offset to less than 5°. Therefore the cross section properties used are satisfactory for steady rotation.

7.0 NON-STEADY ROTATION

The analysis so far has not accounted for the effects of variable angular velocity components. Non-steady rotation will occur during spin-up, and if the vehicle undergoes wobble. It is recalled from (1) and (2) that

$$\ddot{q}_x + \Omega_x(t) q_x = f_x \qquad (15)$$

and

$$\ddot{q}_z + \Omega_z(t) q_z = f_z \qquad (16)$$

where

$$\Omega_x(t) = \frac{K_x}{m} - (\omega_y^2 + \omega_z^2)$$

and

$$\Omega_z(t) = \frac{K_z}{m} - (\omega_x^2 + \omega_y^2)$$

In the non-steady case, the coefficients Ω_x and Ω_z are time dependent. It is well known that simple periodic variations in Ω_x and Ω_z can lead to unbounded deflections. The exact nature of the time dependence of Ω_x and Ω_z requires the solution of the coupled ballast beam-vehicle system of equations.

An alternative approach which avoids the necessity of introducing the vehicle equations would be to obtain, from existing simulations, time histories of ω and $\dot{\omega}$. In the spin-up case, these would depend on the thruster pulsing. These histories can be introduced into equations (15) and (16) and solutions obtained through computer simulations employing the Continuous System Simulation Language (CSSL). Some general results might be obtained from basic theorems on conditions for stability of equations of the type in (15) and (16).[5] Further study in this direction is planned.

Structural and viscous damping have not been included in the analysis for reasons of simplicity. Introducing viscous damping into the ballast beam system is a possible method of increasing regions of stability. Based on available data[6], it does not appear that STEM structural damping alone will be sufficient.

8.0 SUMMARY

The form of equations (1) and (2) for deflection of the ballast beams identify the coefficient of the displacement term as the cause of a possible structural instability (unbounded deflections). This coefficient determines the character of the restoring force and zero, negative, and certain time varying restoring forces result in instabilities.

There are several cases for which stability of ballast beam deflections should be analyzed. The case of steady state rotation has been analyzed here and we can conclude for the Skylab configuration that instability will not arise. Steady rotation does not impose any requirement on beam stiffness when the spin axis and geometric z-axis are within 5°.

Cases of non-steady rotation, including wobble and spin-up, require the solution of equations (1) and (2) coupled with vehicle equations to obtain the most accurate estimate of required stiffness. However, good estimates of required stiffness can probably be obtained by employing rigid body vehicle rates as input into the ballast beam equations. It seems reasonable to expect that for very low wobble rates or spin-up rates, stability boundaries for these cases of non-steady rotation should not differ greatly from the boundaries for steady rotation in Figure 4. Future analysis will determine the importance of the non-steady conditions of spin-up and wobble in imposing stiffness requirements.

1022-RJR-cf

R. J. Ravera

Attachments
References
Figures 1-4

REFERENCES

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6. F. P. J. Remroft, "Storable Tubular Extendible Element - A Unique Machine Element," Machine Design, December 9, 1969, pp. 156-165.

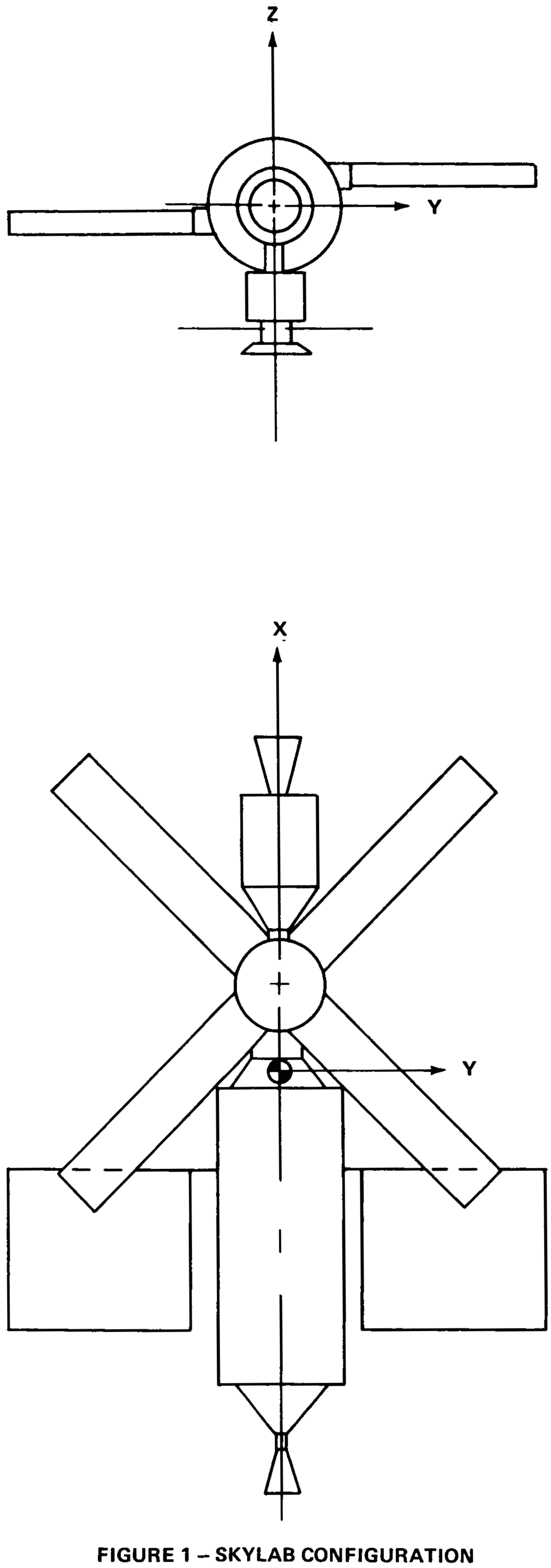

FIGURE 1 – SKYLAB CONFIGURATION

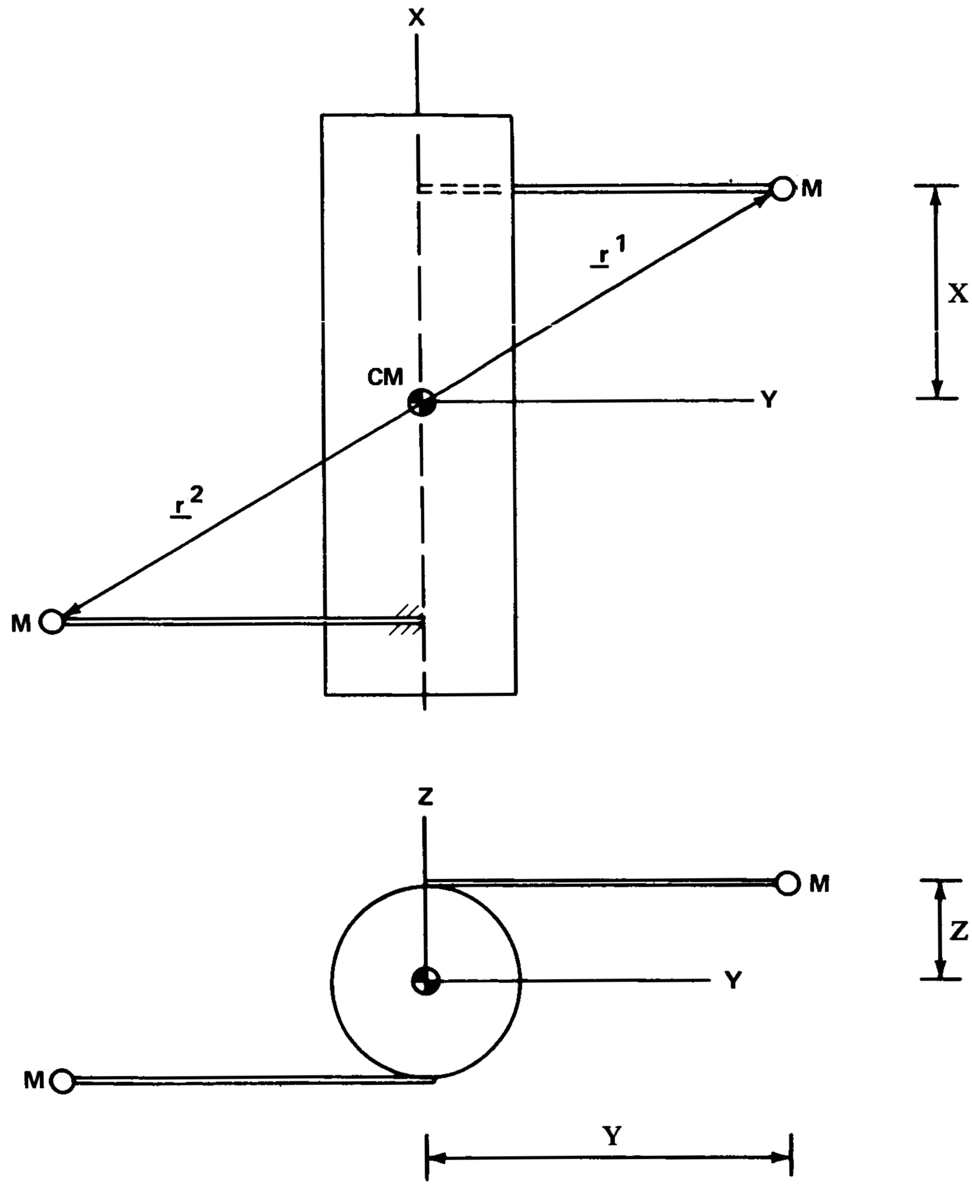

FIGURE 2 – VEHICLE – BALLAST BEAM CONFIGURATION

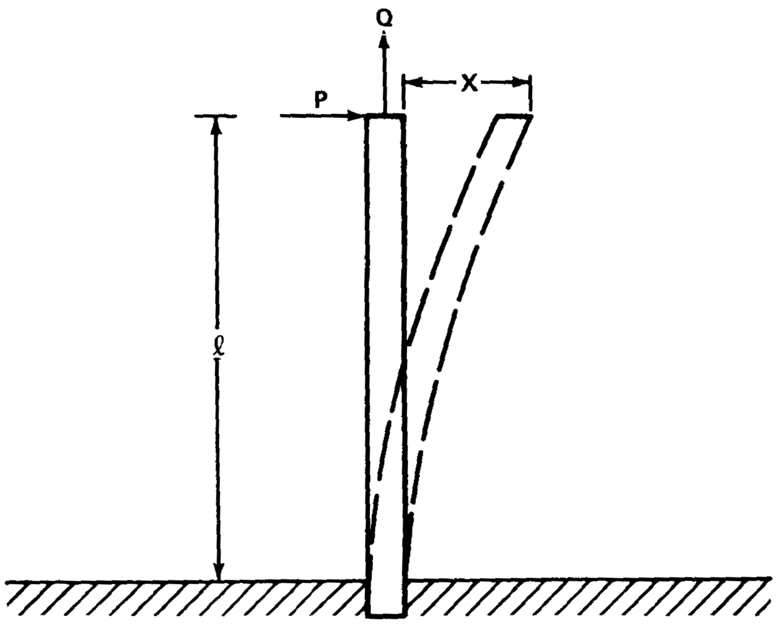

FIGURE 3 – AXIAL AND TRANSVERSE LOADS ON CANTILEVER BEAM

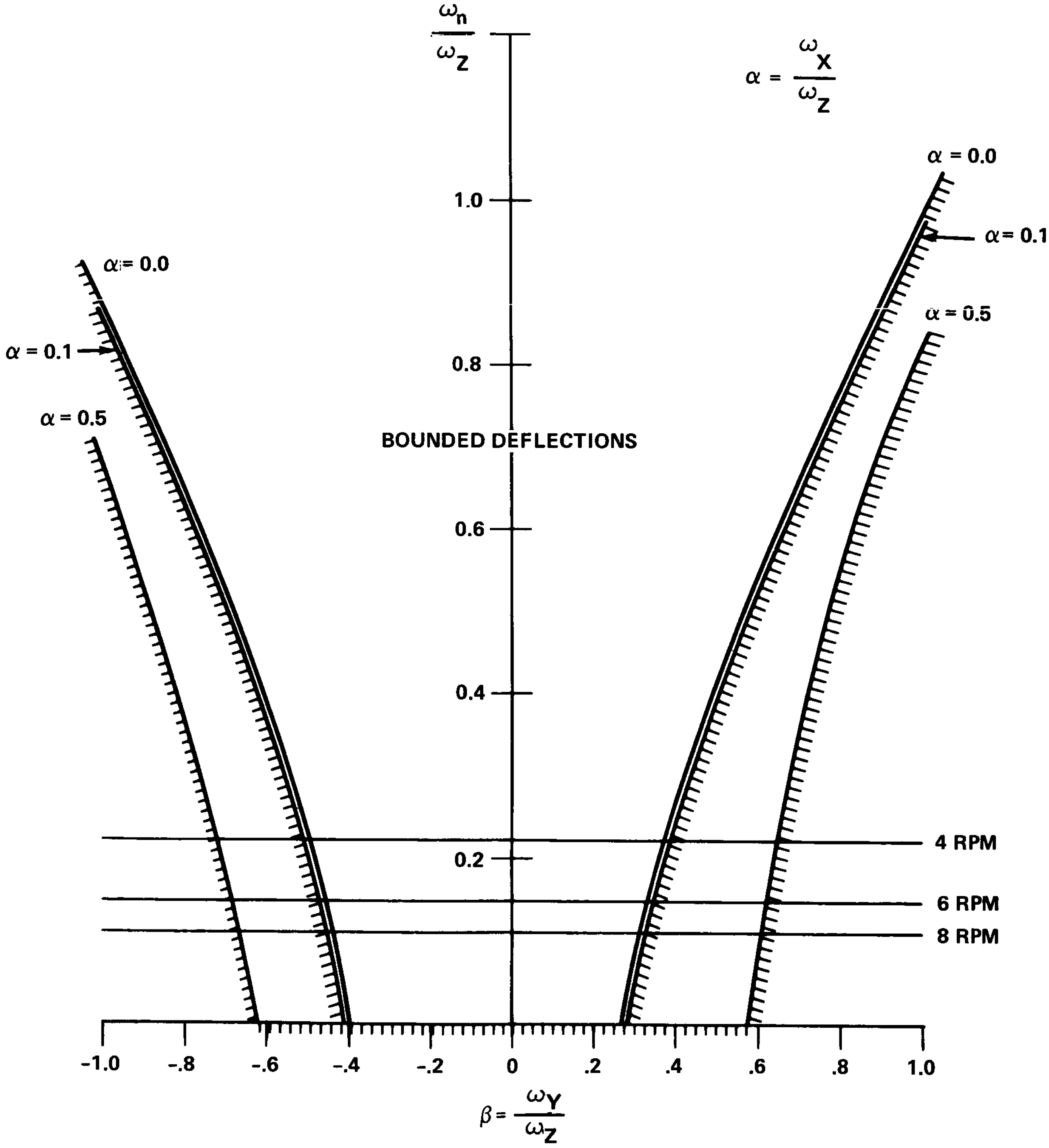

FIGURE 4 - REQUIRED BEAM NATURAL FREQUENCY FOR STEADY - STATE ROTATION